University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 459: Communications I

Fall 2005

# Problem Set 8

## Bandpass Random Processes, Signal-to-Noise Ratios, Noise Analysis in Analog Modulation Schemes

**Issued:** Thursday, Nov. 17th. **Due:** Thursday, December 1st (beginning of lecture).

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**Reading from Lathi:** Chapter 11, Section 11.5; Chapter 12, Sections 12.1–12.3.

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### Problem 8.1

Problems 11.5-1 and 11.5-2 from Lathi, p. 530.

### Problem 8.2

(a) Problem 11.5-3 from Lathi, p. 530.

(b) Consider the problem in part (a) except that $S_{XX}(\omega)$ and $H(\omega)$ are "switched." In other words, $H(\omega)$ is given by the plot in Figure P11.5-3(a) and $S_{XX}(\omega)$ is given by the plot in Figure P11.5-3(b). Does your answer from part (a) change? If so, determine the power spectral densities and the average power of the in-phase and quadrature-phase components of the resulting output process. If there is no change compared to part (a), explain why.

### Problem 8.3 (Optional)

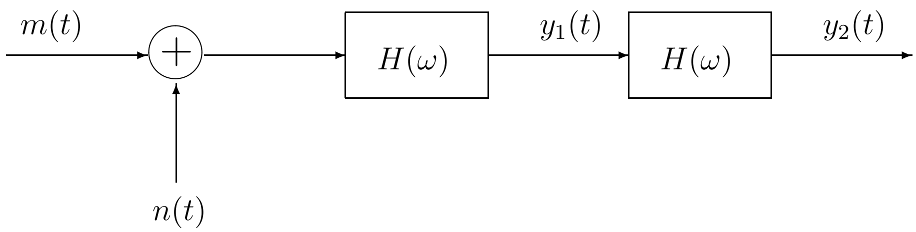
Consider a bandpass Gaussian noise process $N(t)$ with zero mean and power spectral density

$$S_{NN}(\omega) = \begin{cases} \frac{N_0}{2}\,, & \omega_c - 2\pi B \le \omega \le \omega_c + 2\pi B\,, \quad -\omega_c - 2\pi B \le \omega \le -\omega_c + 2\pi B\,, \\ 0\,, & \text{otherwise.} \end{cases}$$

Find the probability density function of a sample of the envelope of $n(t)$, i.e., find $f_R(r)$ of the random variable $R = c(t)$, where $c(t)$ is the envelope of the band-limited random process $N(t)$.

### Problem 8.4

The message signal $m(t) = \cos(\omega_m t)$ gets corrupted by additive white Gaussian noise $N(t)$ with zero mean and power spectral density $S_{NN}(\omega) = 10^{-1}$. The resulting signal gets filtered by the cascade of the two filters shown below.




The frequency response of the filters is given by

$$H_1(\omega) = \begin{cases} 1, & \omega_m - 2\pi \text{ rad/s} < |\omega| < \omega_m + 2\pi \text{ rad/s} \ , \\ 0, & \text{otherwise.} \end{cases} \qquad H_2(\omega) = \begin{cases} 1 - \frac{|\omega|}{2\pi \times 10}, & |\omega| < 2\pi \times 10 \text{ rad/s} \ , \\ 0, & \text{otherwise.} \end{cases}$$

(a) For this part, assume that $\omega_m = 2\pi \times 5$ rad/s.

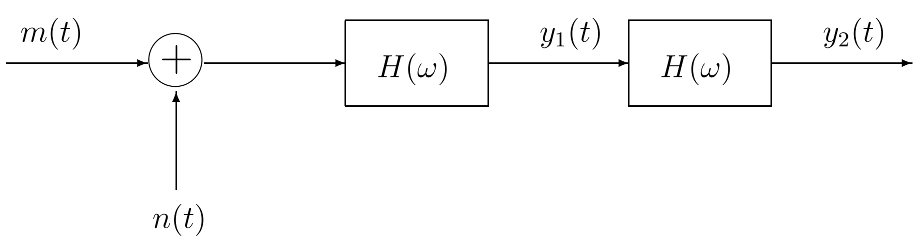
(i) Find the signal to noise ratio at the output $y_1(t)$ of filter $H_1(\omega)$.

(ii) Find the signal to noise ratio at the output $y_2(t)$ of filter $H_2(\omega)$.

(b) Repeat part (a) for $\omega_m = 2\pi \times 9$ rad/s.

**Problem 8.5 (Optional)**

The message signal $m(t) = \cos(\omega_m t)$ gets corrupted by additive white Gaussian noise $N(t)$ with zero mean and power spectral density $S_{NN}(\omega) = 10^{-1}$. The resulting signal gets filtered by the cascade of two identical filters with frequency response $H(\omega)$ as shown below. Assume that $\omega_m = 2\pi \times 6$ rad/s.




The frequency response of the filter is given by

$$H(\omega) = \begin{cases} 1 - \frac{|\omega|}{2\pi \times 10}, & |\omega| < 2\pi \times 10 \text{ rad/s} \ , \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the signal to noise ratio at the output $y_1(t)$.

(b) Find the signal to noise ratio at the output $y_2(t)$.

**Problem 8.6 (Optional)**

The signal $m(t) = A_m \cos(\omega_m t)$ (where $A_m$ and $\omega_m$ are constants) is corrupted by additive white Gaussian noise. More specifically, the corrupted signal $x(t)$ is given by

$$x(t) = m(t) + n(t) ,$$

where $n(t)$ is a sample path of a white Gaussian random process $N(t)$ with zero mean and power spectral density $S_{NN}(\omega) = \frac{N_0}{2}$. Find an expression for the output signal-to-noise ratio after the signal $x(t) = m(t) + n(t)$ is applied to an LTI filter with impulse response $h(t) = e^{-t}u(t)$.

**Problem 8.7**

Problems 12.1-1 and 12.2-1 from Lathi, pp. 572–573.

**Problem 8.8**

Problems 12.2-4 and 12.3-1 from Lathi, pp. 573–574.

**Problem 8.9**

Consider an FM modulation scheme where the message signal $m(t)$ has bandwidth $W = 2\pi \times 8$ rad/s and has power $P = 1/2$. We are required to transmit this signal via a channel with available bandwidth $B = 2\pi \times 60$ rad/s and attenuation 40 dB. The channel noise is additive and white with a power spectral density of $\frac{N_0}{2} = 10^{-12}$.

(a) If it is desirable to have an SNR of at least 40 dB at the receiver output, what is the minimum required transmitter power and the corresponding modulation index?

(b) How does your answer in part (a) change if it is desirable to have an SNR of at least 60 dB at the receiver output?

**Problem 8.10**

The sinusoidal message signal $m(t) = A_m \cos(\omega_m t)$ is frequency modulated so that the transmitted signal $s(t)$ is given by

$$\begin{aligned} s(t) &= A_c \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\tau)d\tau\right] \\ &= A_c \cos\left[\omega_c t + \beta \sin \omega_m t\right] , \end{aligned}$$

where $\beta = \dfrac{k_f A_m}{\omega_m}$. The following parameters are given $\omega_c = 20$ Mrad/s and $A_m = 10$ V.

(a) Let $k_f = 6$ Krad/s/V; specify the range of possible $\omega_m$ such that the bandwidth of the resulting $s(t)$ is below 200 Krad/s.

(b) Signal $s(t)$ goes through a communication channel that adds noise to it so that the received signal $r(t)$ is given by $r(t) = s(t) + n(t)$, where $n(t)$ is a sample path from a white Gaussian random process $N(t)$ with power spectral density $S_{NN}(\omega) = \frac{N_0}{2}$. Signal $r(t)$ is then demodulated using a standard FM demodulation scheme like the one we studied in class. Assuming that the bandwidth of $s(t)$ is required to be at most 200 Krad/s and that the input signal to noise ratio satisfies $SNR_i \equiv \frac{A_c^2 \times 2\pi}{2N_0\omega_m} = 10$, find the range of $\omega_m$ such that the signal to noise ratio at the output of the FM demodulator is at least $\frac{3\times 10^5}{2}$.