ECE 362 Problem Set 7 Solutions

7.1 **5.1** **a** **i** For any given possible subscript *a* of f_1 or f_2, then...

If *a* is a subscript in both functions, then if exactly *a* inputs are 1, then f_1 and f_2 evaluate to 1 and $f_1+f_2=1$.

If *a* is a subscript in exactly one function, then if exactly *a* inputs are 1, then either f_1 or f_2 evaluates to 1 and $f_1+f_2=1$.

If *a* is a subscript in neither function, then if exactly *a* inputs are 1, then f_1 and f_2 evaluate to 0 and $f_1+f_2=0$.

Since the result is a constant for each of the three possibilities, f_1+f_2 is symmetric.

ii Same as above, except...

both	$1 \bullet 1=1$	constant
one	$0 \bullet 1=0$	constant
	$1 \bullet 0=0$	
none	$0 \bullet 0=0$	constant

so $f_1 f_2$ is symmetric

iii

both	$1 \oplus 1=1$	constant
one	$0 \oplus 1=0$	constant
	$1 \oplus 0=0$	
none	$0 \oplus 0=0$	constant

so $f_1 \oplus f_2$ is symmetric

b **i** For any possible subscript *a* of f_1...

If *a* is a subscript of f_1, then $f_1+f_2=1$ – constant

If *a* is not a subscript of f_1, then $f_1+f_2=f_2$

So, for f_1+f_2 to be symmetric, f_2 must be symmetric with respect to input combinations where *a* inputs are 1, but *a* is not a subscript of f_1.

ii For any possible subscript *a* of f_1...

If *a* is a subscript of f_1, then $f_1 f_2=f_2$

If *a* is not a subscript of f_1, then $f_1 f_2=0$ – constant

So, for $f_1 f_2$ to be symmetric, f_2 must be symmetric with respect to input combinations where *a* inputs are 1, and *a* is a subscript of f_1.

iii For any possible subscript *a* of f_1...

If *a* is a subscript of f_1, then $f_1 \oplus f_2 = f_2'$

If *a* is not a subscript of f_1, then $f_1 \oplus f_2 = f_2$

So, for $f_1 \oplus f_2$ to be symmetric, f_2 must be symmetric. Since this forms a contradiction with the requirement that f_2 must not be symmetric, there are no conditions which will cause $f_1 \oplus f_2$ to be symmetric.

7.2 5.2 a Constructive Proof:

For any given a_i $1 \le i \le k$, the presence of this subscript indicates that if exactly a_i variables are equal to 1, the left-hand side is equal to 1. In order for the right-hand side to also equal 1 with the input values being the same, exactly a_i literals of the right-hand side must be 1. Since the literals are all complements, this means that exactly a_i variables must be equal to 0. Since there are n variables, this means $n-a_i$ variables must be equal to 1.

(An analogous argument applies for numbers which are not among the a_i.)

So, the statement is true if $b_i = n-a_i$ for all $1 \le i \le k$.

b $S_{0,1,4}(w,x,y,z)$

5.3 a For each possible subscript....

0: $A=0 \Rightarrow A'S_0=1$, so $f_1=1$

1: $A=0 \Rightarrow A'S_1=1$, so $f_1=1$; $A=1 \Rightarrow AS_0=1$, so $f_1=1$

2: $A=0 \Rightarrow A'S_2$ required, but not present, so $f_1=0$; $A=1 \Rightarrow AS_1$ required, but not present, so $f_1=0$

3: $A=0 \Rightarrow A'S_3$ required, but not present, so $f_1=0$; $A=1 \Rightarrow AS_2$ required, but not present, so $f_1=0$

4: $A=0 \Rightarrow A'S_4=1$, so $f_1=1$; $A=1 \Rightarrow AS_3=1$, so $f_1=1$

5: $A=1 \Rightarrow AS_4=1$, so $f_1=1$

So, $f_1=S_{0,1,4,5}(A,B,C,D,E)$

b $f_3 = A'S_{0,1,2}(B,C,D,E) + AS_{0,1,4}(B,C,D,E)$

For each possible subscript...

0: $A=0 \Rightarrow A'S_0=1$, so $f_3=1$

1: $A=0 \Rightarrow A'S_1=1$, so $f_3=1$; $A=1 \Rightarrow AS_0=1$, so $f_3=1$

2: $A=0 \Rightarrow A'S_2=1$, so $f_3=1$; $A=1 \Rightarrow AS_1=1$, so $f_3=1$

3: $A=0 \Rightarrow A'S_3$ required, bot not present, so $f_3=0$; $A=1 \Rightarrow AS_2$ required, but not present, so $f_3=0$

4: $A=0 \Rightarrow A'S_4$ required, but not present, so $f_3=0$; $A=1 \Rightarrow AS_3$ required, but not present, so $f_3=0$

5: $A=1 \Rightarrow AS_4=1$, so $f_3=1$

$f3=S_{0,1,2,5}(A,B,C,D,E)$

7.3 **5.6** **c** $S_{1,3,4}(x_1',x_2',x_3',x_4',x_5')S_{2,3,4,5}(x_1,x_2,x_3,x_4,x_5)$

$= S_{1,2,4}(x_1,x_2,x_3,x_4,x_5)S_{2,3,4,5}(x_1,x_2,x_3,x_4,x_5)$

$= S_{2,4}(x_1,x_2,x_3,x_4,x_5)$

7.4 **5.7** **a** $f = x \oplus y \oplus c = x'y'c + x'yc' + xy'c' + xyc$

wrt x: $f_x = y'c' + yc$ $f_{x'} = y'c + yc'$
$f_x f_{x'} = (y'c' + yc)(y'c + yc') = 0$ – mixed

wrt y: $f_y = x'c' + xc$ $f_{y'} = x'c + xc'$
$f_y f_{y'} = 0$ – mixed

wrt c: $f_c = x'y' + xy$ $f_{c'} = x'y + xy'$
$f_c f_{c'} = 0$ – mixed

not unate

d

yz

f		00	01	11	10
wx	00			1	1
	01		1	1	1
	11		1		
	10		1	1	

$f = w'z + xy'z + wx'z$

wrt w: $f_w = xy'z + x'z$ $\quad f_{w'} = z + xy'z = z$

$f_w f_{w'} = (xy'z + x'z)z = xy'z + x'z = f_w$

$f_w + f_{w'} = xy'z + x'z + z = z = f_{w'}$

$f_w \Rightarrow f_{w'}$ – negative

wrt x: $f_x = w'z + y'z$ $\quad f_{x'} = w'z + wz = z$

$f_x f_{x'} = (w'z + y'z)z = w'z + y'z = f_x$

$f_x + f_{x'} = w'z + y'z + z = z = f_{x'}$

$f_x \Rightarrow f_{x'}$ – negative

wrt y: $f_y = w'z + wx'z$ $\quad f_{y'} = w'z + xz + wx'z$

$f_y f_{y'} = (w'z + wx'z)(w'z + xz + wx'z) = w'z + wx'z = f_y$

$f_y + f_{y'} = w'z + wx'z +$ ~~$w'z$~~ $+ xz +$ ~~$wx'z$~~ $= f_{y'}$

$f_y \Rightarrow f_{y'}$ – negative

wrt z: $f_z = w' + xy' + wx'$ $\quad f_{x'} = 0$

$f_z f_{z'} = 0 = f_{z'}$

$f_z + f_{z'} = w'\ xy'\ wx' = f_z$

$f_{z'} \Rightarrow f_z$ – positive

unate

7.5 **5.12** **a** $f(w,x,y,z) = wx + xyz$

w	x	y	z	f	
0	0	0	0	0	
0	0	0	1	0	~~$a_z < T$~~
0	0	1	0	0	~~$a_y < T$~~
0	0	1	1	0	~~$a_y + a_z < T$~~
0	1	0	0	0	~~$a_x < T$~~
0	1	0	1	0	$a_x + a_z < T$
0	1	1	0	0	$a_x + a_y < T$
0	1	1	1	1	$a_x + a_y + a_z \geq T$
1	0	0	0	0	~~$a_w < T$~~
1	0	0	1	0	~~$a_w + a_z < T$~~
1	0	1	0	0	~~$a_w + a_y < T$~~
1	0	1	1	0	$a_w + a_y + a_z < T$
1	1	0	0	1	$a_w + a_x \geq T$
1	1	0	1	1	~~$a_w + a_x + a_z \geq T$~~
1	1	1	0	1	~~$a_w + a_x + a_y \geq T$~~
1	1	1	1	1	~~$a_w + a_x + a_y + a_z \geq T$~~

Assume all coeffs are positive and delete redundant inequalities...

$a_c + a_z < T$
$a_x + a_y < T$
$a_w + a_y + a_z < T$
$a_x + a_y + a_z \geq T$
$a_w + a_x \geq T$

Combine these and get...

$a_x > a_w > a_z$
$a_x > a_w > a_y$
$a_x > a_y + a_z$

So, assign $a_w = 2$, $a_x = 3$, $a_y = 1$, $a_z = 1$

$T > a_x + a_z = 4$
$T > a_x + a_y = 4$
$T > a_w + a_y + a_z = 4$
$T \leq a_x + a_y + a_z = 5$
$T \leq a_w + a_x =5$

So, assign T = 5

d For f(w’,x,y,z), start with the same assignments as above (except now use w’)...

$a_{w'} = 2$, $a_x = 3$, $a_y = 1$, $a_z = 1$

From this, we can assign $a_w = -2$. Checking the inequalities shows that T can be assigned as T = 3.

7.6 5.17 a

f (wx \ yz)	00	01	11	10
00		1	1	
01		1	1	
11		1	1	
10	1		1	

Restricting the analysis to 2-variable decompositions, it can be seen that the columns of the K-map above meet the decomposition criteria, so one possible decomposition is f(wx',y,z).

f (wy \ xz)	00	01	11	10
00		1	1	
01		1	1	
11		1	1	
10	1		1	

Since this map looks exactly the same as above, the decomposition looks similar: f(wy',x,z).

f (wz \ xy)	00	01	11	10
00				
01	1	1	1	1
11		1	1	1
10	1			

Again, the same pattern emerges (but, this time in the rows), so there is another similar decomposition: f(x'y',w,z)