ECE 362 Problem Set 5 Solutions

5.1 **a** f = a'bce' + a'c'de + ade' + ab'de + bde
= a'bce' + a'c'de + ade' + ~~ab'de~~ + bde
+ bcde' + b'c'de + ade + ab'd + a'bcd + abd
= a'bce' + ~~a'c'de~~ + ~~ade'~~ + bde + ~~bcde'~~ + ~~b'c'de~~ + ~~ade~~ + ~~ab'd~~ + ~~a'bcd~~ + ~~abd~~
+ c'de + ~~bcde~~ + ~~bc'de~~ + bcd + ~~acde'~~ + ~~ac'de~~ + ad + ~~a'bde~~ + ~~ab'c'd~~ + ~~abcd~~
= a'bce' + bde + c'de + bcd + ad
+ ~~bcde'~~ + ~~a'bcd~~
= a'bce' + bde + c'de + bcd + ad

OR, same method, different look:

Iteration 1:

a	b	c	d	e
0	1	1	-	0
0	-	0	1	1
1	-	-	1	0
~~1~~	~~0~~	~~-~~	~~1~~	~~1~~
-	1	-	1	1
-	1	1	1	0
-	0	0	1	1
1	-	-	1	1
1	0	-	1	-
0	1	1	1	-
1	1	-	1	-

Iteration 2:

a	b	c	d	e
0	1	1	-	0
~~0~~	~~-~~	~~0~~	~~1~~	~~1~~
~~1~~	~~-~~	~~-~~	~~1~~	~~0~~
-	1	-	1	1
~~-~~	~~1~~	~~1~~	~~1~~	~~0~~
~~-~~	~~0~~	~~0~~	~~1~~	~~1~~
~~1~~	~~-~~	~~-~~	~~1~~	~~1~~
~~1~~	~~0~~	~~-~~	~~1~~	~~-~~
~~0~~	~~1~~	~~1~~	~~1~~	~~-~~
~~1~~	~~1~~	~~-~~	~~1~~	~~-~~
-	-	0	1	1
~~-~~	~~1~~	~~1~~	~~1~~	~~1~~
~~-~~	~~1~~	~~0~~	~~1~~	~~1~~
-	1	1	1	-
~~1~~	~~-~~	~~1~~	~~1~~	~~0~~
~~1~~	~~-~~	~~0~~	~~1~~	~~1~~
1	-	-	1	-
~~0~~	~~1~~	~~-~~	~~1~~	~~1~~
~~1~~	~~0~~	~~0~~	~~1~~	~~-~~
~~1~~	~~1~~	~~1~~	~~1~~	~~-~~

Iteration 3:

a	b	c	d	e
0	1	1	-	0
-	1	-	1	1
-	-	0	1	1
-	1	1	1	-
1	-	-	1	-
~~-~~	~~1~~	~~1~~	~~1~~	~~0~~
~~0~~	~~1~~	~~1~~	~~1~~	~~-~~

Iteration 4:

a	b	c	d	e
0	1	1	-	0
-	1	-	1	1
-	-	0	1	1
-	1	1	1	-
1	-	-	1	-

So, the complete sum is:

f = a'bce' + bde + c'de + bcd + ad

b f = uxyz' + vxz + v'yz + w'xyz + w'xz'
= uxyz' + vxz + v'yz + ~~w'xyz~~ + w'xz'
+ xyz + uvxy + vw'x + uv'xy + ~~v'w'xy~~ + ~~uw'xy~~ + w'xy
= ~~uxyz'~~ + vxz + v'yz + w'xz' + xyz + ~~uvxy~~ + vw'x + ~~uv'xy~~ + w'xy
+ ~~uxyz~~ + uxy + ~~w'xyz~~ + ~~uw'xy~~
= vxz + v'yz + w'xz' + xyz + vw'x + w'xy + uxy
+ ~~w'xyz~~ + ~~v'w'xy~~
= vxz + v'yz + w'xz' + xyz + vw'x + w'xy + uxy

OR, same method, different look:

Iteration 1:

u	v	w	x	y	z
1	-	-	1	1	0
-	1	-	1	-	1
-	0	-	-	1	1
~~-~~	~~-~~	~~0~~	~~1~~	~~1~~	~~1~~
-	-	0	1	-	0
-	-	-	1	1	1
1	1	-	1	1	-
-	1	0	1	-	-
1	0	-	1	1	-
~~-~~	~~0~~	~~0~~	~~1~~	~~1~~	~~-~~
~~1~~	~~-~~	~~0~~	~~1~~	~~1~~	~~-~~
-	-	0	1	1	-

Iteration 2:

u	v	w	x	y	z
~~1~~	~~-~~	~~-~~	~~1~~	~~1~~	~~0~~
-	1	-	1	-	1
-	0	-	-	1	1
-	-	0	1	-	0
-	-	-	1	1	1
~~1~~	~~1~~	~~-~~	~~1~~	~~1~~	~~-~~
-	1	0	1	-	-
~~1~~	~~0~~	~~-~~	~~1~~	~~1~~	~~-~~
-	-	0	1	1	-
~~1~~	~~-~~	~~-~~	~~1~~	~~1~~	~~1~~
1	-	-	1	1	-

Iteration 3:

u	v	w	x	y	z
-	1	-	1	-	1
-	0	-	-	1	1
-	-	0	1	-	0
-	-	-	1	1	1
-	1	0	1	-	-
-	-	0	1	1	-
1	-	-	1	1	-

So, the complete sum is:

f = vxz + v’yz + w’xz’ + xyz + vw’x + w’xy + uxy

c f = t’wx’ + twy + uwx + uxy’ + vxz + uwx’y + wx’yz
= t’wx’ + twy + uwx + uxy’ + vxz + ~~uwx’y~~ + ~~wx’yz~~
+ wx’y + t’uw + uwy + ~~uwyz~~ + ~~t’uwy’~~ + t’vwz + uvwyz + vwyz + ~~tuwx~~
= t’wx’ + twy + uwx + uxy’ + vxz + wx’y + t’uw + uwy + t’vwz + vwyz
+ ~~wx’y~~ + ~~uwy~~ + ~~t’uwy’~~ + ~~t’vwz~~ + ~~vwyz~~ + ~~tuwy~~ + ~~uwx~~ + ~~uwxz~~
= t’wx’ + twy + uwx + uxy’ + vxz + wx’y + t’uw + uwy + t’vwz + vwyz

OR, same method, different look:

Iteration 1:

t	u	v	w	x	y	z
0	-	-	1	0	-	-
1	-	-	1	-	1	-
-	1	-	1	1	-	-
-	1	-	-	1	0	-
-	-	1	-	1	-	1
~~-~~	~~1~~	~~-~~	~~1~~	~~0~~	~~1~~	~~-~~
~~-~~	~~-~~	~~-~~	~~1~~	~~0~~	~~1~~	~~1~~
-	-	-	1	0	1	-
0	1	-	1	-	-	-
-	1	-	1	-	1	-
~~-~~	~~1~~	~~-~~	~~1~~	~~-~~	~~1~~	~~1~~
~~0~~	~~1~~	~~-~~	~~1~~	~~-~~	~~0~~	~~-~~
0	-	1	1	-	-	1
~~-~~	~~1~~	~~1~~	~~1~~	~~-~~	~~1~~	~~1~~
-	-	1	1	-	1	1
~~1~~	~~1~~	~~-~~	~~1~~	~~1~~	~~-~~	~~-~~

Iteration 2:

t	u	v	w	x	y	z
0	-	-	1	0	-	-
1	-	-	1	-	1	-
-	1	-	1	1	-	-
-	1	-	-	1	0	-
-	-	1	-	1	-	1
-	-	-	1	0	1	-
0	1	-	1	-	-	-
-	1	-	1	-	1	-
0	-	1	1	-	-	1
-	-	1	1	-	1	1
~~-~~	~~1~~	~~1~~	~~1~~	~~1~~	~~-~~	~~1~~

So, the complete sum is:

f = t’wx’ + twy + uwx + uxy’ + vxz + wx’y + t’uw + uwy + t’vwz + vwyz

5.2 **6.16** **a** As suggested on the newsgroup, we will rewrite the function as:

$f = x_1 f_1(x_2, x_3, x_4, x_5, x_6, x_7) + x_1' f_2(x_2, x_3, x_4, x_5, x_6, x_7)$

where,

$f_1(x_2, x_3, x_4, x_5, x_6, x_7) = \Sigma(0, 5, 16, 32, 48, 53, 61)$
$f_2(x_2, x_3, x_4, x_5, x_6, x_7) = \Sigma(0, 5, 16, 21, 32, 37, 45, 48, 53, 61)$

These are functions of 6 variables, so use K-maps to simplify

f_1, $x_2=0, x_3=0$

x_4x_5 \ x_6x_7	00	01	11	10
00	1			
01		1		
11				
10				

f_1, $x_2=0, x_3=1$

x_4x_5 \ x_6x_7	00	01	11	10
00	1			
01				
11				
10				

f_1, $x_2=1, x_3=0$

x_4x_5 \ x_6x_7	00	01	11	10
00	1			
01				
11				
10				

f_1, $x_2=1, x_3=1$

x_4x_5 \ x_6x_7	00	01	11	10
00	1			
01		1		
11		1		
10				

So, $f_1 = x_4'x_5'x_6'x_7' + x_2'x_3'x_4'x_5x_6'x_7 + x_2x_3x_5x_6'x_7$

Now, f_2:

f_2, $x_2=0, x_3=0$

x_4x_5 \ x_6x_7	00	01	11	10
00	1			
01		1		
11				
10				

f_2, $x_2=0, x_3=1$

x_4x_5 \ x_6x_7	00	01	11	10
00	1			
01		1		
11				
10				

f_2, $x_2=1, x_3=0$

x_4x_5 \ x_6x_7	00	01	11	10
00	1			
01		1		
11		1		
10				

f_2, $x_2=1, x_3=1$

x_4x_5 \ x_6x_7	00	01	11	10
00	1			
01		1		
11		1		
10				

So, $f_2 = x_4'x_5'x_6'x_7' + x_4'x_5x_6'x_7 + x_2x_5x_6'x_7$

$f = x_1f_1 + x_1'f_2$
$= x_1x_4'x_5'x_6'x_7' + x_1x_2'x_3'x_4'x_5x_6'x_7 + x_1x_2x_3x_5x_6'x_7$
$+ x_1'x_4'x_5'x_6'x_7' + x_1'x_4'x_5x_6'x_7 + x_1'x_2x_5x_6'x_7$

Now, use this SOP to solve for the PIs:

Iteration 1:

x_1	x_2	x_3	x_4	x_5	x_6	x_7
~~1~~	~~-~~	~~-~~	~~0~~	~~0~~	~~0~~	~~0~~
~~1~~	~~0~~	~~0~~	~~0~~	~~1~~	~~0~~	~~1~~
~~1~~	~~1~~	~~1~~	~~-~~	~~1~~	~~0~~	~~1~~
~~0~~	~~-~~	~~-~~	~~0~~	~~0~~	~~0~~	~~0~~
0	-	-	0	1	0	1
0	1	-	-	1	0	1
-	-	-	0	0	0	0
-	0	0	0	1	0	1
~~-~~	~~1~~	~~1~~	~~0~~	~~1~~	~~0~~	~~1~~
-	1	1	-	1	0	1

Iteration 2:

x_1	x_2	x_3	x_4	x_5	x_6	x_7
0	-	-	0	1	0	1
0	1	-	-	1	0	1
-	-	-	0	0	0	0
-	0	0	0	1	0	1
-	1	1	-	1	0	1
~~0~~	~~-~~	~~0~~	~~0~~	~~1~~	~~0~~	~~1~~

So, the prime implicants are:

$A = x_2'x_3'x_4'x_5x_6'x_7$ (5, 69)
$B = x_1'x_4'x_5x_6'x_7$ (5, 21, 37, 53)
$C = x_1'x_2x_5x_6'x_7$ (37, 45, 53, 61)
$D = x_2x_3x_5x_6'x_7$ (53, 61, 117, 125)
$E = x_4'x_5'x_6'x_7'$ (0, 16, 32, 48, 64, 80, 96, 112)

b

	0	5	16	21	32	37	45	48	53	61	64	69	80	96	112	117	12
A		x										(x)					
B		x		(x)		x			x								
C						x	(x)		x	x							
D									x	x						(x)	(x)
E	(x)		(x)		(x)			(x)			(x)		(x)	(x)	(x)		

All prime implicants are essential, so the (trivial) Petrick expression is ABCDE, and the minimal sum is:

$f = A + B + C + D + E$
$= x_2'x_3'x_4'x_5x_6'x_7 + x_1'x_4'x_5x_6'x_7 + x_1'x_2x_5x_6'x_7 + x_2x_3x_5x_6'x_7 + x_4'x_5'x_6'x_7'$

5.3 $f = A_2B_2' + A_2B_2A_1B_1' + A_2'B_2'A_1B_1' + A_2B_2A_1B_1A_0B_0' + A_2B_2A_1'B_1'A_0B_0'$
$+ A_2'B_2'A_1B_1A_0B_0' + A_2'B_2'A_1'B_1'A_0B_0'$

Iteration 1:

A_2	A_1	A_0	B_2	B_1	B_0
1	-	-	0	-	-
~~1~~	~~1~~	~~-~~	~~1~~	~~0~~	~~-~~
~~0~~	~~1~~	~~-~~	~~0~~	~~0~~	~~-~~
~~1~~	~~1~~	~~1~~	~~1~~	~~1~~	~~0~~
~~1~~	~~0~~	~~1~~	~~1~~	~~0~~	~~0~~
~~0~~	~~1~~	~~1~~	~~0~~	~~1~~	~~0~~
~~0~~	~~0~~	~~1~~	~~0~~	~~0~~	~~0~~
-	1	-	0	0	-
-	1	1	0	1	0
-	0	1	0	0	0
1	-	1	1	0	0
0	-	1	1	0	0
1	1	-	-	0	-
1	1	1	-	1	0
1	0	1	-	0	0
1	1	1	1	-	0
0	1	1	0	-	0

Iteration 2:

A_2	A_1	A_0	B_2	B_1	B_0
1	-	-	0	-	-
-	1	-	0	0	-
~~-~~	~~1~~	~~1~~	~~0~~	~~1~~	~~0~~
~~-~~	~~0~~	~~1~~	~~0~~	~~0~~	~~0~~
~~1~~	~~-~~	~~1~~	~~1~~	~~0~~	~~0~~
~~0~~	~~-~~	~~1~~	~~1~~	~~0~~	~~0~~
1	1	-	-	0	-
~~1~~	~~1~~	~~1~~	~~-~~	~~1~~	~~0~~
~~1~~	~~0~~	~~1~~	~~-~~	~~0~~	~~0~~
~~1~~	~~1~~	~~1~~	~~1~~	~~-~~	~~0~~
~~0~~	~~1~~	~~1~~	~~0~~	~~-~~	~~0~~
-	1	1	0	-	0
-	-	1	1	0	0
~~-~~	~~1~~	~~1~~	~~1~~	~~0~~	~~0~~
~~-~~	~~1~~	~~1~~	~~0~~	~~0~~	~~0~~
~~-~~	~~0~~	~~1~~	~~1~~	~~0~~	~~0~~
-	-	1	0	0	0
~~1~~	~~-~~	~~1~~	~~0~~	~~0~~	~~0~~
1	-	1	-	0	0
~~0~~	~~-~~	~~1~~	~~0~~	~~0~~	~~0~~
~~1~~	~~1~~	~~1~~	~~-~~	~~0~~	~~0~~
0	1	1	-	0	0
0	0	1	-	0	0
1	1	1	-	-	0
~~1~~	~~1~~	~~1~~	~~0~~	~~-~~	~~0~~

Iteration 3:

A_2	A_1	A_0	B_2	B_1	B_0
1	-	-	0	-	-
-	1	-	0	0	-
1	1	-	-	0	-
-	1	1	0	-	0
~~-~~	~~-~~	~~1~~	~~1~~	~~0~~	~~0~~
~~-~~	~~-~~	~~1~~	~~0~~	~~0~~	~~0~~
~~1~~	~~-~~	~~1~~	~~-~~	~~0~~	~~0~~
~~0~~	~~1~~	~~1~~	~~-~~	~~0~~	~~0~~
~~0~~	~~0~~	~~1~~	~~-~~	~~0~~	~~0~~
1	1	1	-	-	0
~~-~~	~~1~~	~~1~~	~~0~~	~~0~~	~~0~~
~~-~~	~~0~~	~~1~~	~~0~~	~~0~~	~~0~~
~~-~~	~~1~~	~~1~~	~~-~~	~~0~~	~~0~~
~~-~~	~~0~~	~~1~~	~~-~~	~~0~~	~~0~~
~~0~~	~~-~~	~~1~~	~~0~~	~~0~~	~~0~~
~~0~~	~~-~~	~~1~~	~~-~~	~~0~~	~~0~~
-	-	1	-	0	0

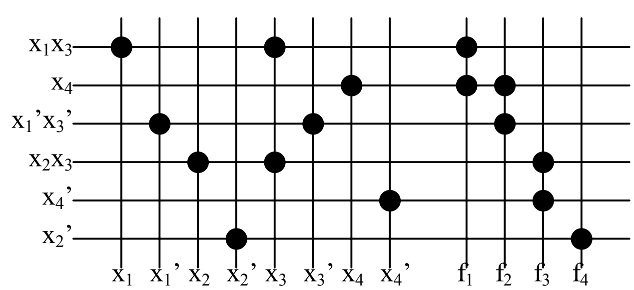
Iteration 4:

A_2	A_1	A_0	B_2	B_1	B_0
1	-	-	0	-	-
-	1	-	0	0	-
1	1	-	-	0	-
-	1	1	0	-	0
1	1	1	-	-	0
-	-	1	-	0	0

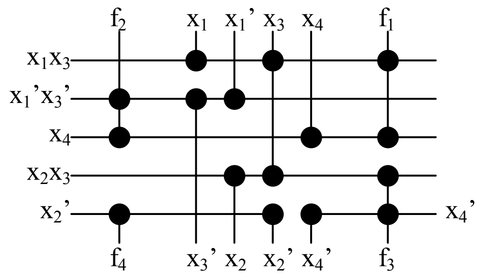
So, the prime implicants are:

A_2B_2'
$A_1B_2'B_1'$
$A_2A_1B_1'$
$A_1A_0B_2'B_0'$
$A_2A_1A_0B_0'$
$A_0B_1'B_0'$

5.4 **6.23** **a**



b Obviously, by using inspection we will never know if we actually have the minimal, so a good solution is acceptable for this problem. Here is one:

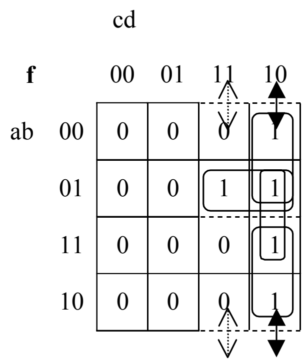


5.5 3.8 a $f = (((a'b')'c(ad)')'(a'cd')')'$
$= (a'b')'c(ad)' + a'cd'$
$= (a + b)c(a' + d') + a'cd'$
$= aa'c + acd' + a'bc + bcd' + a'cd'$

So, the 1-sets are:

[a, a', c] [a, c, d'] [a', b, c] [b, c, d'] [a', c, d']

b



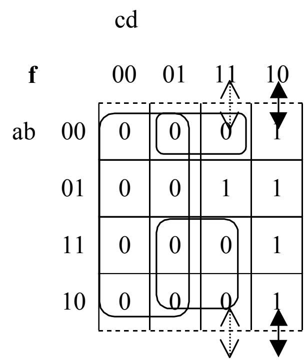
By inspection, the static 1-hazards are (<abcd>):
<0010> - <1010>
By inspection, the static 0-hazards are (<abcd>):
<0011> - <1011>

c $f^D = (((a' + b')' + c + (a + d)')' + (a' + c + d')')'$
$= ((a' + b')' + c\,(a + d)')(a' + c + d')$
$= (ab + c + a'd')(a' + c + d')$
$= aa'b +$ ~~abc~~ $+ abd' +$ ~~$a'c$~~ $+ c +$ ~~cd'~~ $+ a'd' +$ ~~$a'cd'$~~
$= aa'b + abd' + c + a'd'$

So, the 0-sets are:

[a, a', b] [a, b, d'] [c] [a', d']

d



By inspection, the static 1-hazards are (<abcd>):
<0010> - <1010>
By inspection, the static 0-hazards are (<abcd>):
<0011> - <1011>