ECE 362 Problem Set 10 Solutions

10.1 **8.2** **b** Excitation Equations:

$Q_1^+ = D$ <CLOCK>↑
$= ((X' + Q_1'')Q_2')'$ <CLOCK>↑
$= ((X' + Q_1)' + Q_2)$ <CLOCK>↑
$= (XQ_1' + Q_2)$ <CLOCK>↑

$J = X$
$K = Q_1 \oplus X$
$= Q_1'X + Q_1X'$

$Q_2^+ = (J'K'Q_2 + JK' + JKQ_2')$ <CLOCK>↑
$= (X'(Q_1'X + Q_1X')'Q_2 + X(Q_1' + Q_1X')' + X(Q_1'X + Q_1X')Q_2'$ <CLOCK>↑
$= (X'(Q_1'X)'(Q_1X')'Q_2 + X(Q_1'X)'(Q_1X')' + XQ_1'Q_2')$ <CLOCK>↑
$= (X'(Q_1 + X')(Q_1' + X)Q_2 + X(Q_1 + X')(Q_1' + X) + XQ_1'Q_2')$ <CLOCK>↑
$= (X'(Q_1X + Q_1'X')Q_2 + X(Q_1X + Q_1'X') + XQ_1'Q_2')$ <CLOCK>↑
$= (X'Q_1'Q_2 + XQ_1 + XQ_1'Q_2')$ <CLOCK>↑
$= (X'Q_1'Q_2 + XQ_1 + XQ_2')$ <CLOCK>↑

$Z = (Q_1Q_2')' = Q_1' + Q_2$

Transition Table:

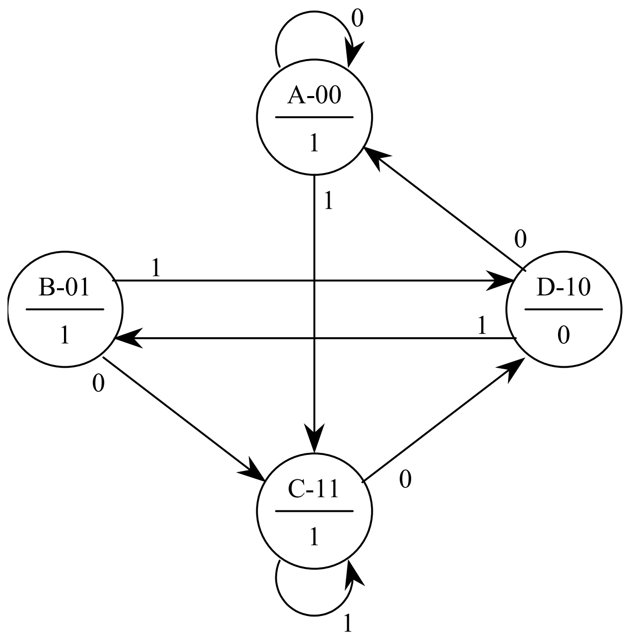
		<CLOCK>↑				
Q_1	Q_2	**0**		**1**		**Z**
0	0	0	0	1	1	1
0	1	1	1	1	0	1
1	1	1	0	1	1	1
1	0	0	0	0	1	0

$Q_1^+Q_2^+$

State/Output Table:

			<CLOCK>↑		
Q_1	Q_2	**s**	**0**	**1**	**Z**
0	0	A	A	C	1
0	1	B	C	D	1
1	1	C	D	C	1
1	0	D	A	B	0

State/Output Diagram:



10.2 8.4 b Excitation Equations:

$Q_1^+ = 0<x_1>\downarrow + 1<x_2>\downarrow + 1<x_3>\downarrow$

$Q_2^+ = 1<x_1>\downarrow + 0<x_2>\downarrow + 1<x_3>\downarrow$

Since the x_1, x_2 and x_3 pulses are non-overlapping, and since Q_1 and Q_2 can only change on a negative edge of x_1, x_2 or x_3, Q_1 and Q_2 cannot change when $x_1=1$, $x_2=1$, or $x_3=1$. Thus, the equation for Q_3 can be simplified by writing as a response to positive edges on x_1, x_2 and x_3.

$Q_3^+ = Q_1Q_2<x_1>\uparrow + Q_1'<x_2>\uparrow + Q_2'<x_3>\uparrow$

$Z = Q_3$

Transition Table:

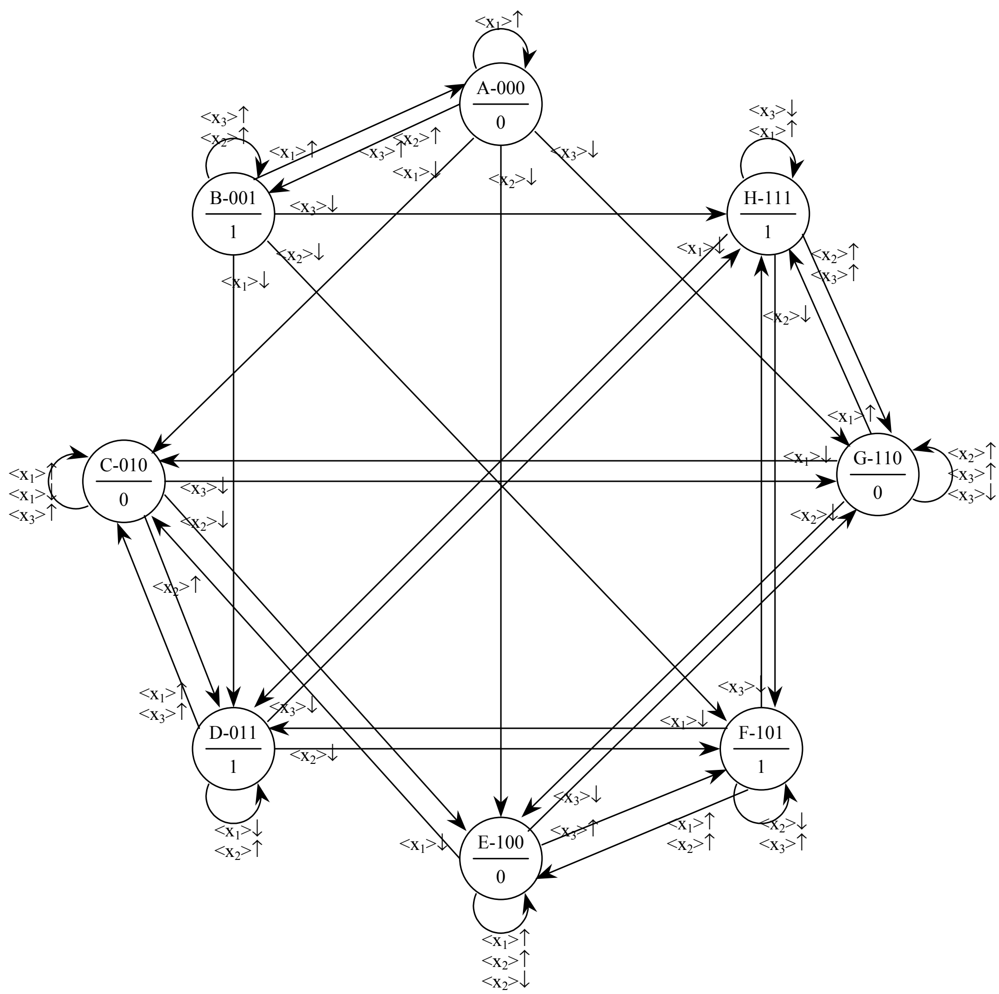
Q_1	Q_2	Q_3	$<x_1>\uparrow$	$<x_1>\downarrow$	$<x_2>\uparrow$	$<x_2>\downarrow$	$<x_3>\uparrow$	$<x_3>\downarrow$	Z
0	0	0	000	010	001	100	001	110	0
0	0	1	000	011	001	101	001	111	1
0	1	0	010	010	011	100	010	110	0
0	1	1	010	011	011	101	010	111	1
1	0	0	100	010	100	100	101	110	0
1	0	1	100	011	100	101	101	111	1
1	1	0	111	010	110	100	110	110	0
1	1	1	111	011	110	101	110	111	1

$Q_1^+Q_2^+Q_3^+$

c Flow Table (Which in this case, is the same as the state table):

Q_1	Q_2	Q_3	s	$<x_1>\uparrow$	$<x_1>\downarrow$	$<x_2>\uparrow$	$<x_2>\downarrow$	$<x_3>\uparrow$	$<x_3>\downarrow$	Z
0	0	0	A	A	C	B	E	B	G	0
0	0	1	B	A	D	B	F	B	H	1
0	1	0	C	C	C	D	E	C	G	0
0	1	1	D	C	D	D	F	C	H	1
1	0	0	E	E	C	E	E	F	G	0
1	0	1	F	E	D	E	F	F	H	1
1	1	0	G	H	C	G	E	G	G	0
1	1	1	H	H	D	G	F	G	H	1

d State Diagram:



10.3 The question is whether the minimum state required to detect the condition is finite. Consider the state required to detect the condition if the length of the string is known to be n. The key thing to note is that the ones and zeros can occur in any order.

If n is not divisible by 4, the output is always 0, so no state is needed.

If n is divisible by 4, then state must be kept. There are three ways of keeping the state. First, the number of zeros can be counted. There are at most $3n/4$ zeros in a sequence that meets the condition, so this requires $3n/4$ states. Second, the number of ones can be counted. There are at most $n/4$ ones in a sequence that meets the condition, so this requires $n/4$ states. Third, the difference $d = 3N_{zeros} - N_{ones}$ can be tracked. Note that the range of possible values for d is $-n \le d \le 3n$, so this requires $4n$ states. Also, note that all of the methods require an amount of state which is linear in n.

By obvious contradiction, the amount of state needed to determine the condition criteria with n unknown cannot be less than the amount of state needed to determine it with n known. Clearly, then, the amount of state needed is $\Omega(n)$. So, as n approaches infinity (which is possible, because in the problem, n is unknown), the amount of state needed is infinite. Therefore, the statement is true.