University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2002

Problem Set 1

Linear Time-Invariant Systems, Fourier Transform, Hilbert Transform

Issued: Thursday, Sept. 5th. **Due:** Beginning of lecture on Thursday, Sept. 12th.

Reading from Proakis (2nd Edition): Chapter 1 and Chapter 2. Most of this material should be familiar to you from previous courses.

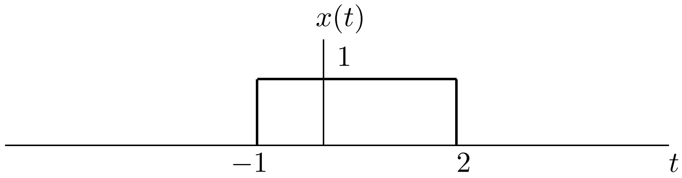
Problem 1.1

(a) Consider an LTI system with input $x(t)$ and output $y(t)$ related through the equation

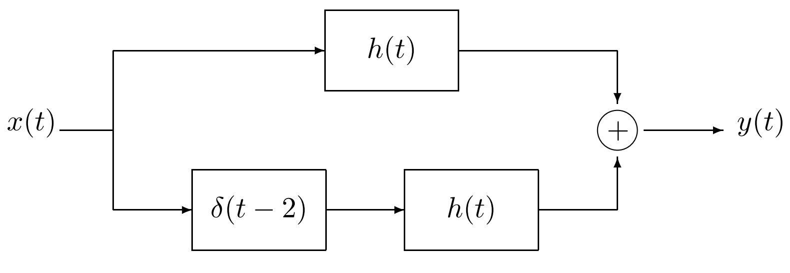
$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 5) d\tau.$$

What is the impulse response $h(t)$ for this system?

(b) Determine the response $y(t)$ of the system in part (a) when the input $x(t)$ is as shown below.



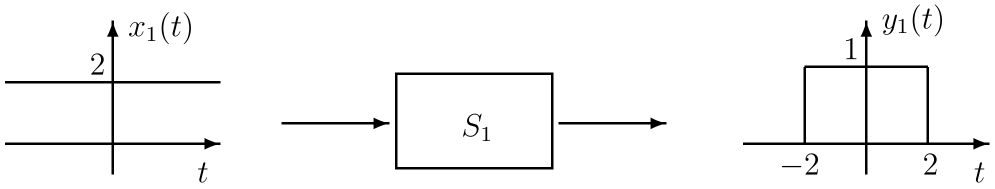
(c) Consider the following interconnection of LTI systems:



Here, $h(t)$ is as in part (a). Determine the output $y(t)$ when the input $x(t)$ is given as in part (b). (Hint: You do *not* need to evaluate a convolution integral.)

Problem 1.2

(a) The following system S_1 (*not necessarily* LTI) is known to have the input-output pair shown below:



Is system S_1 time-invariant? Explain.

(b) A *nonlinear* system S_2 is known to be time-invariant. Is the output $y_2(t)$ of system S_2 guaranteed to be periodic in t when input $x_2(t) = \cos(2\pi f_0 t)$ is applied? Explain.

Problem 1.3

Show that the Fourier transform of signal $x(t)$ can be written as

$$X(f) = \int_{-\infty}^{\infty} x(t)\cos(2\pi f t)dt - j\int_{-\infty}^{\infty} x(t)\sin(2\pi f t)dt\ .$$

Using the above, show that if $x(t)$ is an even function of t, then

$$X(f) = 2\int_{0}^{\infty} x(t)\cos(2\pi f t)dt\ .$$

Also, show that a signal $x(t)$ that is a *real and even* function of t has Fourier transform $X(f)$ that is a *real and even* function of f.

Problem 1.4

(a) **(Optional)** Problem 2.10 from Proakis (2nd Edition), p. 60.

(b) Problem 2.21 from Proakis (2nd Edition), p. 62.

Problem 1.5 (Optional)

Show that $\int_{-\infty}^{+\infty} \text{sinc}^2(kx)dx = 1/k$.

Problem 1.6

A signal $x(t)$ is applied to a square-law device whose output $y(t)$ is defined by

$$y(t) = x^2(t) \ .$$

If the spectrum of $x(t)$ is limited to the frequency interval $-W \leq f \leq W$, show that the spectrum of $y(t)$ is limited to $-2W \leq f \leq 2W$.

Problem 1.7 (Optional)

A linear time-invariant system S is known to be stable. Show that, if the input $x(t)$ of S has finite energy, then the output $y(t)$ also has finite energy. In other words, show that if

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty \ ,$$

then

$$\int_{-\infty}^{+\infty} |y(t)|^2 dt < \infty \ .$$

Problem 1.8

(a) Show that, if signal $g(t)$ has Fourier transform $G(f)$, then the Fourier transform of

$$g'(t) = g(t+T) + g(t-T)$$

is given by

$$G'(f) = 2G(f)\cos(2\pi f T) \ .$$

(b) What is the Fourier transform $G(f)$ of the signal

$$g(t) = \text{rect}(t-4) + \text{rect}(t+4) \ ?$$

(Note that the function $\text{rect}(t)$ is the same as the function $\Pi(t)$ in the textbook).

(c) What is the signal $g(t)$ that has Fourier transform

$$G(f) = \text{rect}(f-4) + \text{rect}(f+4) \ ?$$

Problem 1.9

Find the energy and half-power (3dB) bandwidth of the signal $x(t) = e^{-5t}u(t)$.

Problem 1.10

Consider the following scenario:

- Signal $g_1(t) = 10^3 \text{rect}(10^4 t)$ is applied as input to an ideal low-pass filter with frequency response $H_1(f) = \text{rect}(f/20000)$ to produce output $y_1(t)$.
- Signal $g_2(t) = \delta(t)$ is applied as input to an ideal low-pass filter with frequency response $H_2(f) = \text{rect}(f/10000)$ to produce output $y_2(t)$.
- Outputs $y_1(t)$ and $y_2(t)$ are then multiplied to obtain the final output $y(t) = y_1(t)y_2(t)$.

(a) Find $G_1(f)$ and $G_2(f)$.

(b) Find $h_1(t)$ and $h_2(t)$.

(c) Find $Y_1(f)$ and $Y_2(f)$; also, find the bandwidths of $y_1(t)$, $y_2(t)$ and $y(t)$.

Problem 1.11

Problems 2.46 and 2.47 from Proakis (2nd Edition), p. 67.