University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2000

Problem Set 1

Introduction, Linear Time-Invariant Systems, Fourier Transform

Issued: Thursday, August 31st. **Due:** Beginning of lecture on Thursday, September 7th.

Reading from Lathi: Chapter 1, Chapter 2 (excluding Sections 2.5–2.7) and Chapter 3. The material in Chapter 1 is introductory and we will be revisiting it in more detail during the course of the semester. Most of the material in Chapters 2 and 3 should be already familiar to you.

Reading from Haykin: Chapter 1 and Chapter 2 (Sections 2.1–2.9). Most of the material in Chapter 2 should be familiar to you from previous courses.

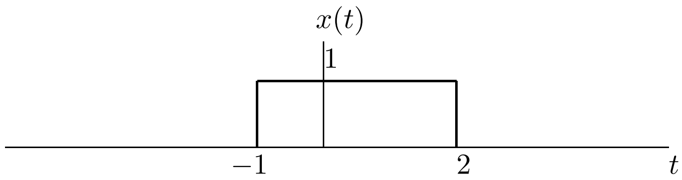
Problem 1.1 (Optional)

(a) Consider an LTI system with input $x(t)$ and output $y(t)$ related through the equation

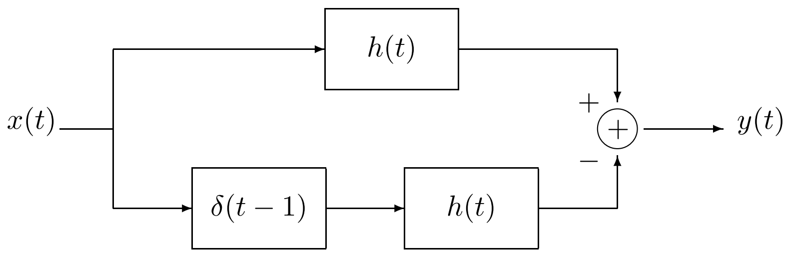
$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 4) d\tau.$$

What is the impulse response $h(t)$ for this system?

(b) Determine the response of this system when the input $x(t)$ is as shown below.



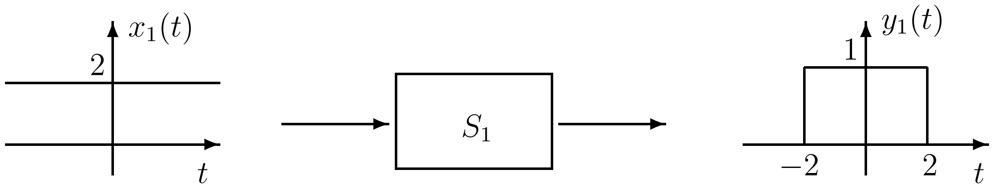
(c) Consider the following interconnection of LTI systems:



Here $h(t)$ is as in part (a). Determine the output $y(t)$ when the input $x(t)$ is given as in part (b). (Hint: You do *not* need to evaluate a convolution integral.)

Problem 1.2

(a) The following system S_1 (*not necessarily* LTI) is known to have the input-output pair shown below:



Is system S_1 time-invariant? Explain.

(b) A *nonlinear* system S_2 is known to be time-invariant. Is the output $y_2(t)$ of system S_2 guaranteed to be periodic in t when input $x_2(t) = \sin(2\pi f_0 t)$ is applied? Explain.

Problem 1.3

Find the energy and half-power (3dB) bandwidth of the signal $x(t) = e^{-3t}u(t)$.

Problem 1.4

Problem 3.1–1 from Lathi, p. 142.

Problem 1.5

Consider the signal $x(t) = A\,\mathrm{rect}\left(\frac{t}{T} - \frac{1}{2}\right)$. Find the Fourier transforms of the even part $x_e(t)$ and the odd part $x_o(t)$ of $x(t)$, defined as

$$\begin{aligned} x_e(t) &= \frac{1}{2}\left[x(t) + x(-t)\right] \ , \\ x_o(t) &= \frac{1}{2}\left[x(t) - x(-t)\right] \ . \end{aligned}$$

Problem 1.6

A signal $x(t)$ is applied to a square-law device whose output $y(t)$ is defined by

$$y(t) = x^2(t) .$$

If the spectrum of $x(t)$ is limited to the frequency interval $-W \leq f \leq W$, show that the spectrum of $y(t)$ is limited to $-2W \leq f \leq 2W$.

Problem 1.7 (Optional)

(a) Find the Fourier transform of the *Gaussian* signal

$$x(t) = Ae^{-t^2/2T^2} , \quad -\infty < t < +\infty .$$

(b) If this signal goes through an LTI system with frequency response

$$H(f) = e^{-\frac{2\pi f}{2B^2}} , \quad -\infty < \omega < +\infty ,$$

what is the output $y(t)$? Simplify the expression for $y(t)$ for $T >> 1/B$.

Problem 1.8

A linear time-invariant system S is known to be stable. Show that, if the input $x(t)$ of S has finite energy, then the output $y(t)$ also has finite energy. In other words, show that if

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty ,$$

then

$$\int_{-\infty}^{+\infty} |y(t)|^2 dt < \infty .$$