ECE 310 **Fall 2000**
UNIVERSITY OF ILLINOIS **Profs. Bresler & Munson**

Test #2
Monday, October 30, 2000

Name: _____ _____

Score: _____

Please ***circle*** the name of your instructor above!

This is a closed-book and closed-notes exam. However, you are permitted two sides of two 8.5" by 11" crib sheets. No calculators may be used. The first ten questions are short-answer. The last four questions are lengthier. On the longer questions, show all of your work for maximum partial credit. Backs of pages may be used for scratch work if necessary.

Good luck!!

(40 pts.) **SHORT ANSWER** *(4 pts. each for Probs. 1-10)*

1. An LSI system has the unit-pulse response $h_n = \cos\left(\pi\sqrt{n}\right)u_n$.

Is the system BIBO stable? YES NO

Why? _____

_____.

2. The input x_n and output y_n of a causal LSI system are related by the difference equation

$$y_n + \frac{3}{2}y_{n-1} - y_{n-2} = x_n.$$

Is the system BIBO stable? YES NO

Why? _____

_____.

3. An LSI system has transfer function

$$H(z) = \frac{z^3 - 3}{\left(z - \frac{1}{2}\right)(z-2)}.$$

a. For which choice of ROC_H is the system BIBO stable?

ROC_H :

b. Is there a choice of ROC_H for which the system is causal but unstable?

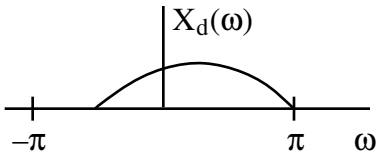
YES NO

4. The causal LSI system with transfer function $H(z) = \dfrac{z^3 - 3}{z - e^{-j\frac{\pi}{3}}}$ is unstable.

Find a <u>real-valued</u> bounded input $\{x_n\}$ such that the output $\{y_n\}$ is unbounded.

$x_n =$

5. Suppose that $\{x_n\}$ has the real-valued DTFT

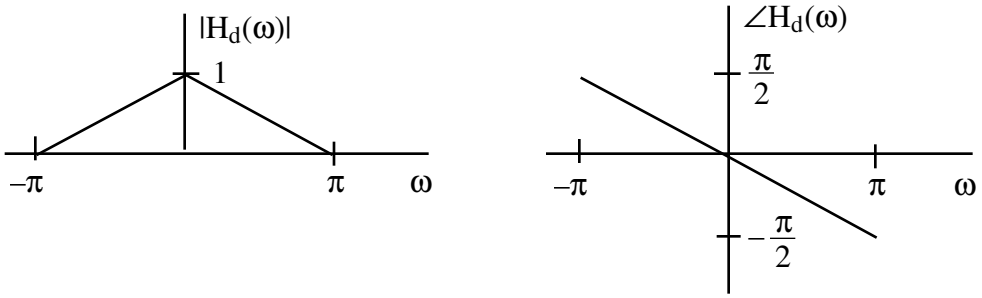


Is $\{x_n\}$ real valued? YES NO

Why or why not? _____

_____.

6. For the LSI system having $|H_d(\omega)|$ and $\angle H_d(\omega)$ below, find y_n if $x_n = \cos\frac{3\pi}{4}n \qquad -\infty < n < \infty$.



$y_n =$ _____

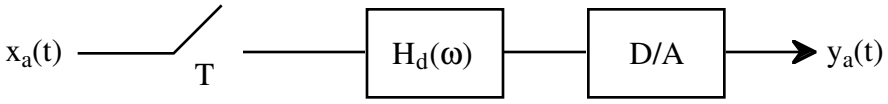
7. Consider the system

$$y_n = \frac{1}{3}y_{n-1} + x_n.$$

If $x_n = \sin\frac{\pi}{2}n \qquad -\infty < n < \infty$, then the amplitude of the sinusoidal output y_n is

_____ .

8. Design the system below to implement an analog LPF with cutoff $\Omega_c = 200\,\pi$ rad/sec. Assume the input is bandlimited to $800\,\pi$ rad/sec.



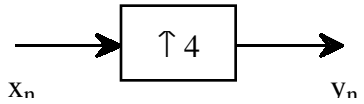
a. The largest T that will prevent aliasing at the sampler is T = _____ .

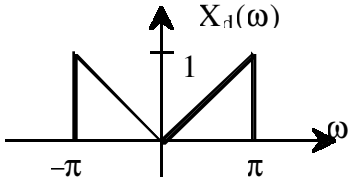
b. For this value of T, the cutoff of the digital filter should be $\omega_c =$ _____ .

9. In the system in Problem 8, it is possible to allow some aliasing at the sampler and still have the system perform like the desired analog LPF.

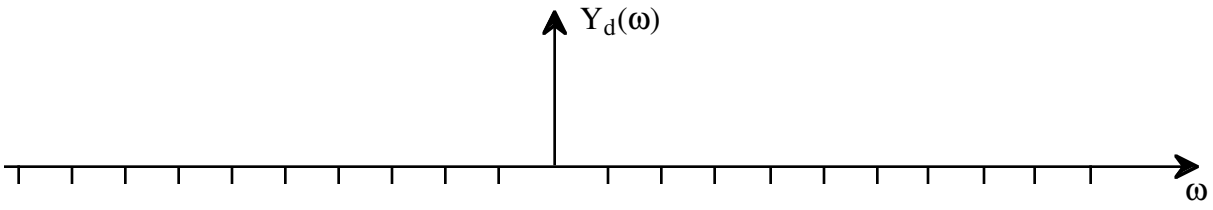
The corresponding largest value of T is $\boxed{}$.

10. Consider the system shown below, with input $\{x_n\}$ having the DTFT $X_d(\omega)$ as shown.





Sketch $Y_d(\omega)$ for $|\omega| \le \pi$. Label your axes and all transition points.

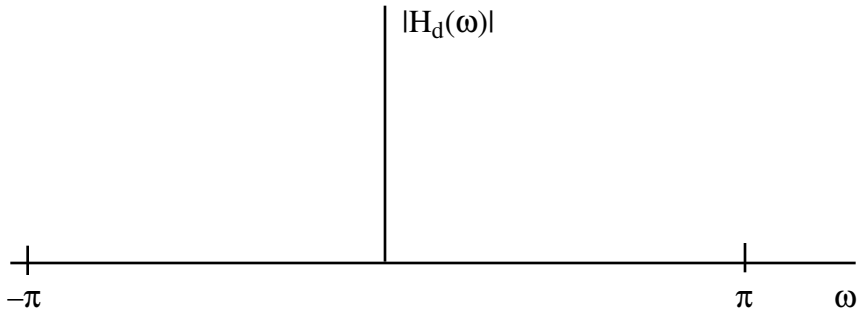


(16 pts.)

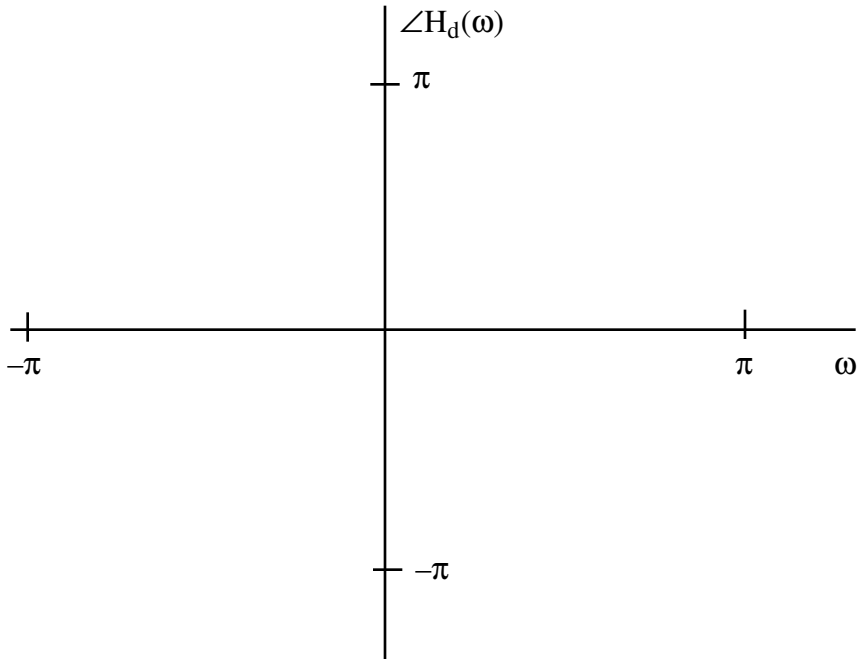
11. A particular causal LSI system has the unit-pulse response

$$\{h_n\}_{n=0}^{\infty} = \{-1, 1, -1, 0, 0, 0, 0, 0, \ldots\}.$$

a. Sketch the system magnitude response $|H_d(\omega)|$. Label $|H_d(0)|$, $|H_d(\pi)|$ and any frequencies where $H_d(\omega) = 0$.

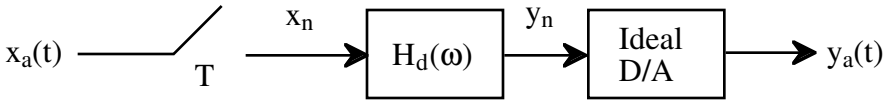


b. Sketch the system phase response.

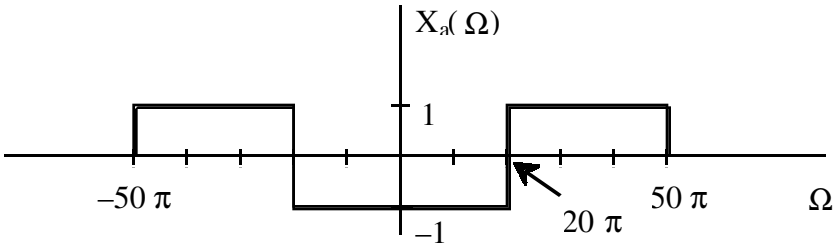


(14 pts.)

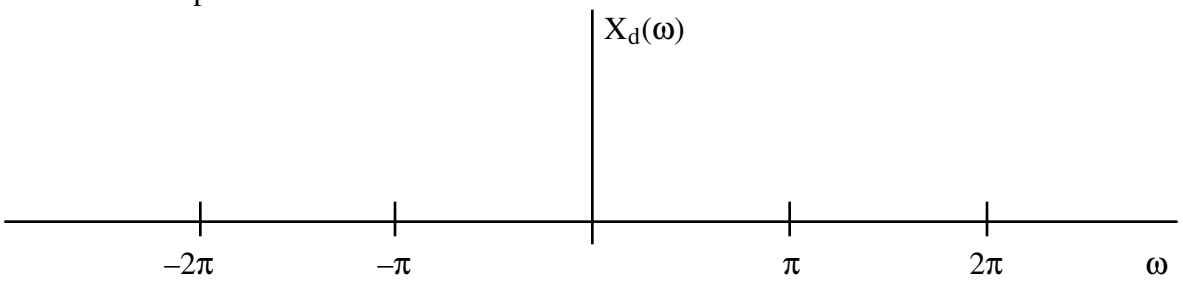
12. Consider

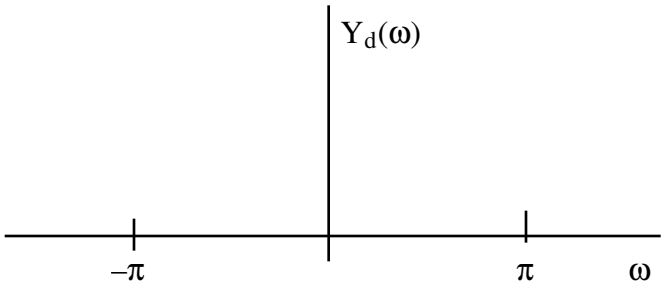


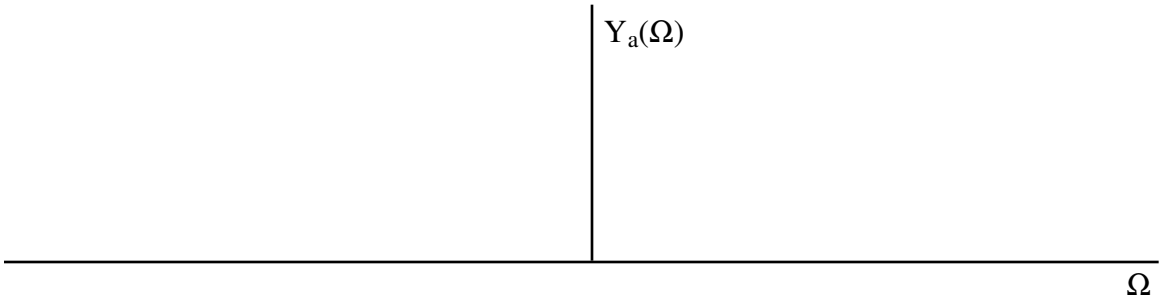
where $T = \frac{1}{40}$, $H_d(\omega)$ is an ideal high-pass filter with $\omega_c = \frac{\pi}{2}$ and



Sketch the spectra indicated below.

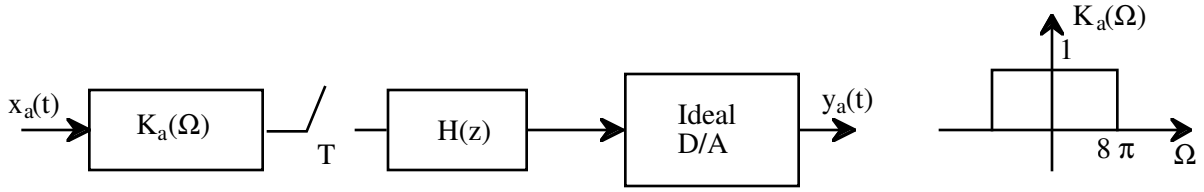






(18 pts.)

13. Consider the system shown below



where $K_a(\Omega)$ is an analog LPF with frequency response as shown.

a. Suppose $H(z) = \dfrac{z-1}{z-\frac{1}{2}}$, and T = 0.1 sec. Is the system, from input $x_a(t)$ to output $y_a(t)$, an LSI system? If LSI, give a closed-form expression for its analog frequency response. If not LSI, explain why.

_____ Yes, LSI

$H_a(\Omega) =$

_____ Not LSI. Why? _____

_____

13. *(continued)*

b. Repeat a. for T = 0.3 sec.

_____ Yes, LSI

$H_a(\Omega) =$

_____ Not LSI. Why? _____

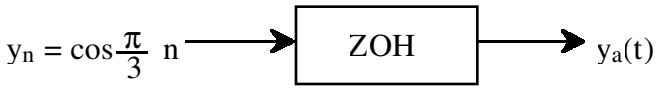
_____

c. Suppose now H(z) = 1, T = 0.3, and $x_a(t) = \cos(5\pi t)$. Find a closed-form expression for $y_a(t)$.

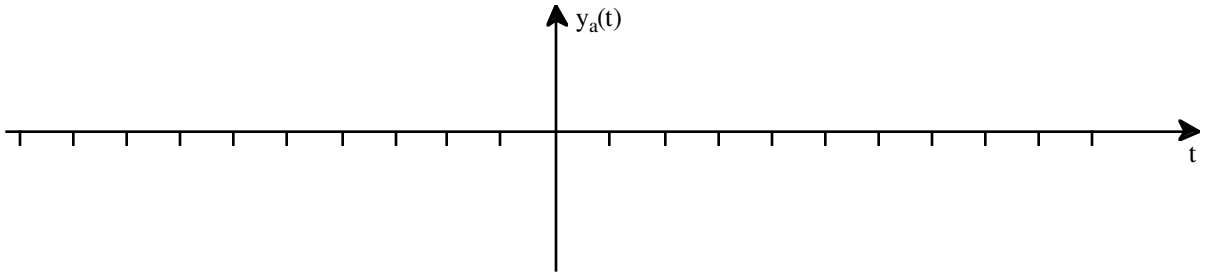
$y_a(t)$

(12 pts.)

14. Consider the system shown below, where the ZOH is operating with time interval T = 01 sec.



a. Sketch the output $y_a(t)$. Make sure to label the axes and all transition points.



b. Sketch the magnitude $|Y_a(\Omega)|$ of the Fourier transform of the output for $|\Omega| \leq 40\ \pi$. Indicate the frequencies of all components.