University of Illinois **Fall 1999**
ECE 310 **Profs. Liang & Werner**

Final Exam

December 14, 1999

Problem	Pts.	Score
1	05	
2	04	
3	04	
4	05	
5	05	
6	06	
7	05	
8	06	
9	05	
10	08	
11	05	
12	12	
13	04	
14	06	
15	10	
16	04	
17	06	
Total	100	

Name ______________________________

Section: 12:00 1:00 PM

Score ____________

Please do not turn this page over until told to do so.

GOOD LUCK!

(5 Pts.)

1. Mark "True" or "False" for the following statements.
   You will receive 1 or -1 point for a correct or incorrect answer, respectively. Negative cummulative scores of this problem will be rounded to zero.

T F a) FIR filters always have generalized linear phase.

T F b) The BL transform $s = \alpha\frac{1-z^{-1}}{1+z^{-1}}$ can map a high-pass analog filter to a low-pass digital filter.

T F c) In windowing design of FIR filters, the rectangular window gives a shorter transition band than the Hamming window.

T F d) In windowing design of FIR filters, the rectangular window gives lower ripples than the Hamming window.

T F e) If an LSI system is BIBO unstable, its unit pulse response $h(n)$ must be unbounded.

(4 Pts.)

2. The system equation of an LSI system is given below:

$$y(n+1) - y(n) + \frac{1}{9}y(n-1) = x(n+1) - \frac{1}{2}x(n)$$

   (a) Sketch the block diagram (direct form I) of the system

   (b) Determine the transfer function of the system.

(4 Pts.)

3. The transfer function of a causal system is given by

$$H(z) = \frac{3z - 2}{z - \frac{1}{2}}$$

Determine the unit pulse response $h(n)$ of the system.

(5 Pts.)

4. The transfer function $H(z)$ of a causal LSI system is given by

$$H(z) = \frac{z - \frac{1}{2}}{(z + \frac{1}{2})(z - \frac{1}{3})}$$

Calculate the output sequence of the system for input $x(n) = 5(\frac{1}{2})^n u(n)$.

(5 Pts.)

5. A two-sided input sequence $x(n)$ having z-transform:

$$X(z) = \frac{z-3}{(z+2)(z-4)}$$

is input to an LSI system with transfer function $H(z)$, producing a two-sided output sequence $y(n)$ having z-transform:

$$Y(z) = \frac{1}{(z+1)(z-4)}.$$

Determine the system's transfer function $H(z)$ and ALL its valid ROCs. For each case, indicate if the system is causal and stable.

(6 Pts.)

6. The unit pulse response of an LSI system is

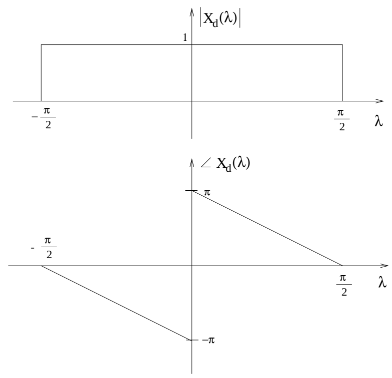
$$h(n) = \begin{cases} n2^{-n}, & \text{for the even values of } n \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Is the system BIBO stable? (Justify your answer.)

(b) Determine the system's transfer function.

(5 Pts.)

7. Let $X_d(\lambda)$ be defined as in the figure below.



Given that

$$\text{DTFT}\left\{\frac{a}{\pi}\text{sinc}(an)\right\} = \begin{cases} 1 & |\lambda| < a \\ 0 & \text{else} \end{cases}$$

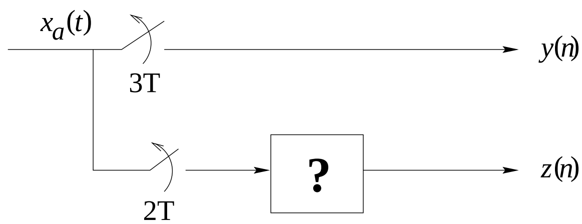
Determine $\text{DTFT}^{-1}\{X_d(\lambda)\}$. Your answer should not contain any complex numbers.

(6 Pts.)

8. The zero-state response of an unknown linear, causal, shift-invariant system for input $x(n) = 2^n u(n)$ is $y(n) = e^{n^2} u(n) + 2^{n-1} u(n-1)$, where $u(n)$ is the unit step function. Determine the unit-pulse response $h(n)$ of the system.

(5 Pts.)

9. Consider the following system consisting of two synchronized ideal A/D convertors. Assume that the input analog signal $x_a(t)$ is bandlimited to $\omega_0 = \pi/(3T)$. Design a digital rate conversion subsystem marked with "?" using down-sampler(s), up-sampler(s), and digital filter(s) as necessary such that $y(n) = z(n)$. Draw a block diagram and determine all the essential parameters of the subsystem.



(8 Pts.)

10. Use the windowing method (with a rectangular window) to design a length-52, GLP, high-pass FIR filter with cutoff frequency $\frac{3\pi}{4}$. Show in detail your design process leading to the unit pulse response of the filter in the form of a sinc function. (Just giving the final unit pulse response of the filter based on the known formula will receive no partial credit!)

(5 Pts.)

11. The bilinear transformation $s = 2(1 - z^{-1})/(1 + z^{-1})$ was applied to an analog protype $H_L(s) = 1/(s^4 + 1/2)$ to design a digital filter. Calculate the (steady-state) response $y(n)$ of the digital filter for input $x(n) = 3\cos(\frac{\pi}{3}n + 45^\circ)$.

(12 Pts.)

12. Let $\{x_n\}_{n=0}^{1} = \{-2, 2\}$, $\{y_n\}_{n=0}^{3} = \{0, -2, 2, 0\}$, and $\{z_n\}_{n=0}^{3} = \{0, -2, 0, 2\}$.

   (a) Calculate $X_d(\lambda)$ and sketch $|X_d(\lambda)|$ and $\angle X_d(\lambda)$ for $|\lambda| < \pi$.

   (b) Calculate $Y_d(\lambda) = \text{DTFT}\{y_n\}$ and $Z_d(\lambda) = \text{DTFT}\{z_n\}$. Express the results in terms of $X_d(\lambda)$.

   (c) Calculate $Y(m) = \text{DFT}\{y_n\}$ and $Z(m) = \text{DFT}\{z_n\}$, where the DFT is length-4. Express the results in terms of $X_d(\lambda)$.

(4 Pts.)

13. Let $\{X_m\}_{m=0}^{31}$ and $X_d(\lambda)$ be the DFT and DTFT of a *real-valued* sequence $\{x_n\}_{n=0}^{31}$. Determine ALL the correct equalities in the following list

    (a) $X_{29} = X_d^*(\frac{3\pi}{16})$
    (b) $X_{29} = X_d(-\frac{3\pi}{16})$
    (c) $X_{29} = X_d(\frac{3\pi}{16})$
    (d) $X_{29} = X_d^*(-\frac{3\pi}{16})$
    (e) $X_{29} = X_d^*(\frac{29\pi}{16})$
    (f) $X_{29} = X_d(-\frac{29\pi}{16})$
    (g) $X_{29} = X_d(\frac{29\pi}{16})$
    (h) $X_{29} = X_d^*(-\frac{29\pi}{16})$

(6 Pts.)

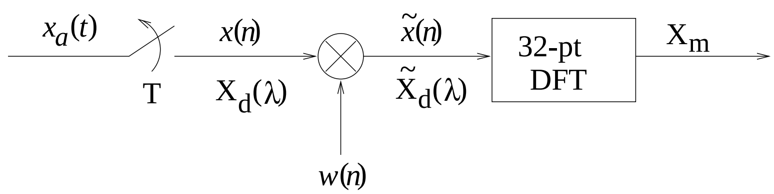
14. Let $\{x_n\}_{n=0}^{4} = \{1, 2, 3, 0, 0\}$ and $\{y_n\}_{n=0}^{4} = \{1, -1, 0, 0, 0\}$.

    (a) Evaluate circular convolution: $\{x_n\}_{n=0}^{4} * \{y_n\}_{n=0}^{4} =$

    (b) Evaluate linear convolution: $\{x_n\}_{n=0}^{4} * \{y_n\}_{n=0}^{4} =$

(10 Pts.)

15. Consider the following system used for DFT spectral estimation.



Let $x_a(t) = 5\cos(2\pi t)$, $T = 1/8$, and $w(n)$ is a length-32 rectangular window.

(a) Determine $W_d(\lambda)$ and sketch $|W_d(\lambda)|$ for $|\lambda| < \pi$ (clearly label your plot).

(b) Determine $X_d(\lambda)$ and sketch $X_d(\lambda)$ for $|\lambda| < \pi$.

(c) Determine and sketch $\{|X_m|\}_{m=0}^{31}$.

(4 Pts.)

16. The following linear convolution

$$\{x_n\}_{n=0}^{46} * \{h_n\}_{n=0}^{32}$$

is to be evaluated using the DFT method. Namely,

$$\{x_n\}_{n=0}^{46} * \{h_n\}_{n=0}^{32} = \mathrm{DFT}^{-1}\{\mathrm{DFT}\{x_n\} \cdot \mathrm{DFT}\{h_n\}\}$$

(a) Determine the minimum number of zeros should be padded to $\{x_n\}$ and $\{h_n\}$, respectively, before the DFTs are applied.

(b) If the DFTs are to be calculated with a radix-2 FFT algorithm, how many zeros should now be padded to $\{x_n\}$ and $\{h_n\}$, respectively.

(6 Pts.)

17. Complete the following signal flow diagram (butterfly structure) of a 4-pt, radix-2, decimation-in-time FFT algorithm. Specify all the connection weights and determine the indexes (a, b, c, and d) of the input signal sequence.