UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 DIGITAL SIGNAL PROCESSING
Spring 1999

EXAM 3

Wednesday, April 21, 1999

- This is a CLOSED BOOK exam, but three $8\frac{1}{2}'' \times 11''$ sheet of notes (both sides) are allowed (no xeroxed sheets).
- NO CALCULATORS are allowed.
- There are 4 problems on the exam, with point values indicated, and not in order of difficulty.
- You must SHOW YOUR WORK to get full credit.
- Please BE NEAT—we can't grade what we can't decipher.
- Don't forget to PUT YOUR NAME ON ALL SHEETS.
- Only this booklet is handed in. NOTHING ELSE WILL BE CONSIDERED.

Problem	Score
1	
2	
3	
4	
Total	

Name: ____

Circle One: LEVINSON SINGER

Problem 1 (10 points)

Consider the system in the figure below. Let $x_a(t)$ be an analog signal bandlimited to $\frac{\pi}{T}$, i.e.,

$$X_a(\Omega) = 0, \quad |\Omega| > \frac{\pi}{T}$$

This signal is sampled at a rate of $1/T$ samples per second, such that $x[n] = x_a(nT)$. The signal $x[n]$ is processed by the discrete-time system with impulse response $h[n]$, such that

$$y[n] = \frac{1}{2}y[n-1] + x[n].$$

Next, the signal $y[n]$ is passed through a zero-order hold (ZOH) device, such that

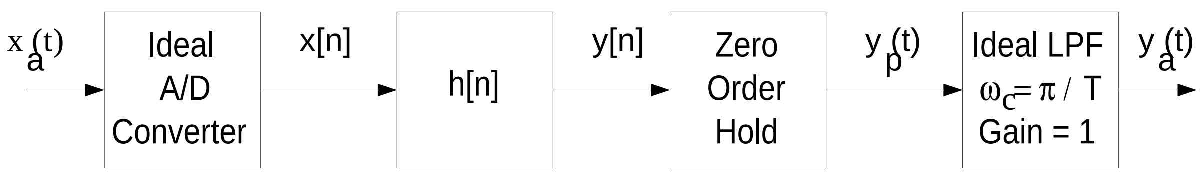
$$y_p(t) = \sum_{k=-\infty}^{\infty} y[k]p(t-kT),$$

and

$$p(t) = \begin{cases} 1, & 0 \leq t \leq T \\ 0, & \text{else.} \end{cases}$$

The signal $y_p(t)$ is then lowpass filtered with an ideal lowpass filter, with frequency response $G_a(\Omega)$,

$$G_a(\Omega) = \begin{cases} 1, & |\Omega| \leq \frac{\pi}{T} \\ 0, & |\Omega| > \frac{\pi}{T}. \end{cases}$$



(a) (2 points) Determine $X_d(\omega)$, the DTFT of $x[n]$, in terms of $X_a(\Omega)$.

(b) (2 points) Determine $Y_d(\omega)$, the DTFT of $y[n]$, in terms of $X_a(\Omega)$.

(c) (3 points) Determine $Y_p(\Omega)$, the Fourier Transform of $y_p(t)$, in terms of $X_a(\Omega)$.

(d) (3 points) Determine $Y_a(\Omega)$, the Fourier Transform of $y_a(t)$, in terms of $X_a(\Omega)$. Also find $H_{eff}(\Omega)$, the effective frequency response of the entire system that takes $x_a(t)$ as input and produces $y_a(t)$ as output.

Problem 2 (10 points)

In this problem, you would like to use the window method to design a causal, finite-length impulse response (FIR) filter, which is an approximation to the ideal highpass filter:

$$D(\omega) = \begin{cases} 0, & |\omega| \le \frac{\pi}{4} \\ 1, & \frac{\pi}{4} < |\omega| < \pi \end{cases}$$

where $d[n]$ is an infinite-length sequence that is symmetric about $n = 0$. Note that your FIR filter will have generalized linear phase.

(a) (2 points) Determine $d[n]$, the impulse response of the desired ideal high pass filter.

(b) (3 points) Let $h[n] = w[n]g[n]$ be the impulse response of your FIR filter, for a rectangular window, with

$$w[n] = \begin{cases} 1, & n = 0, \dots, N-1 \\ 0, & \text{otherwise.} \end{cases}$$

Determine $g[n]$, and also determine $h[n]$, the impulse response of the causal FIR approximation of length N.

(c) (2 points) Make a rough sketch of what you might expect the magnitude $|H_d(\omega)|$ to look like. Include any important features. Also make a detailed, accurate plot of the phase, $\angle\left[H_d(\omega)\right]$ labelling any important features.

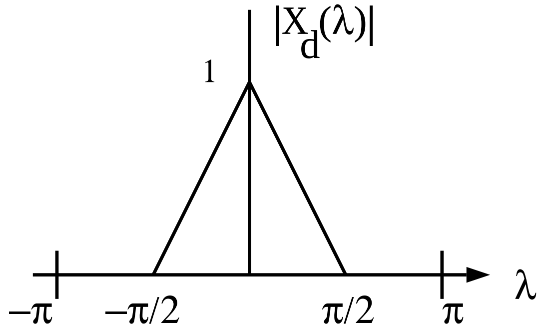
(d) (3 points) Suppose an analog filter with Laplace transform $H_L(s)$ has generalized linear phase, i.e. $H_a(\Omega) = R(\Omega)e^{j(\alpha - M\Omega)}$. A discrete-time filter is then designed by the following mapping,

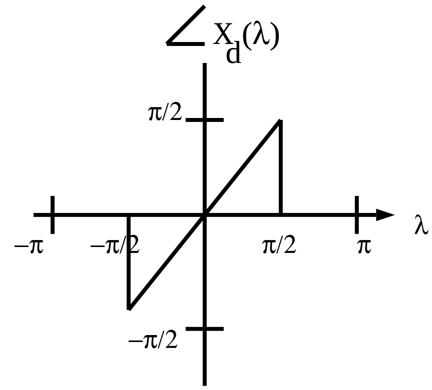
$$H(z) = H_L(s)\Big|_{s = \dfrac{1 - z^{-1}}{1 + z^{-1}}}$$

Discuss whether or not the discrete-time filter with frequency response $H_d(\omega)$ will have generalized linear phase?

Problem 3 (10 points)

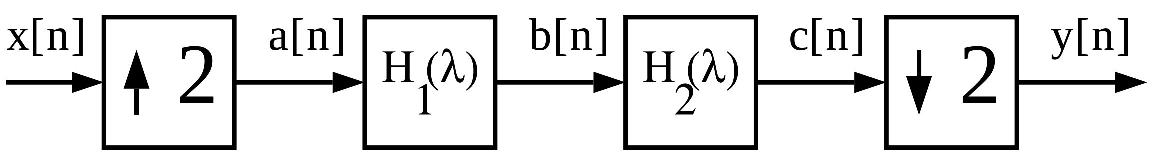
Given a signal $x[n] = [\frac{1}{2}\text{sinc}\left(\frac{\pi}{4}(n+1)\right)]^2$, with discrete-time Fourier transform shown below:



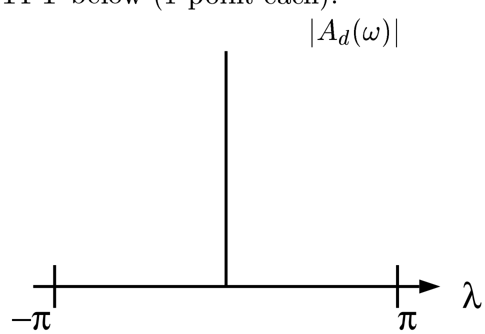


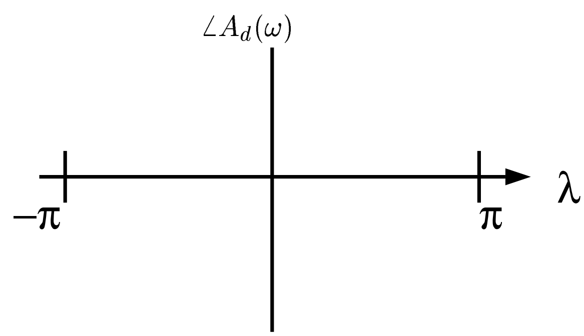
Let $x[n]$ be the input to the system shown below, where $H_1(\omega)$, and $H_2(\omega)$ are given by

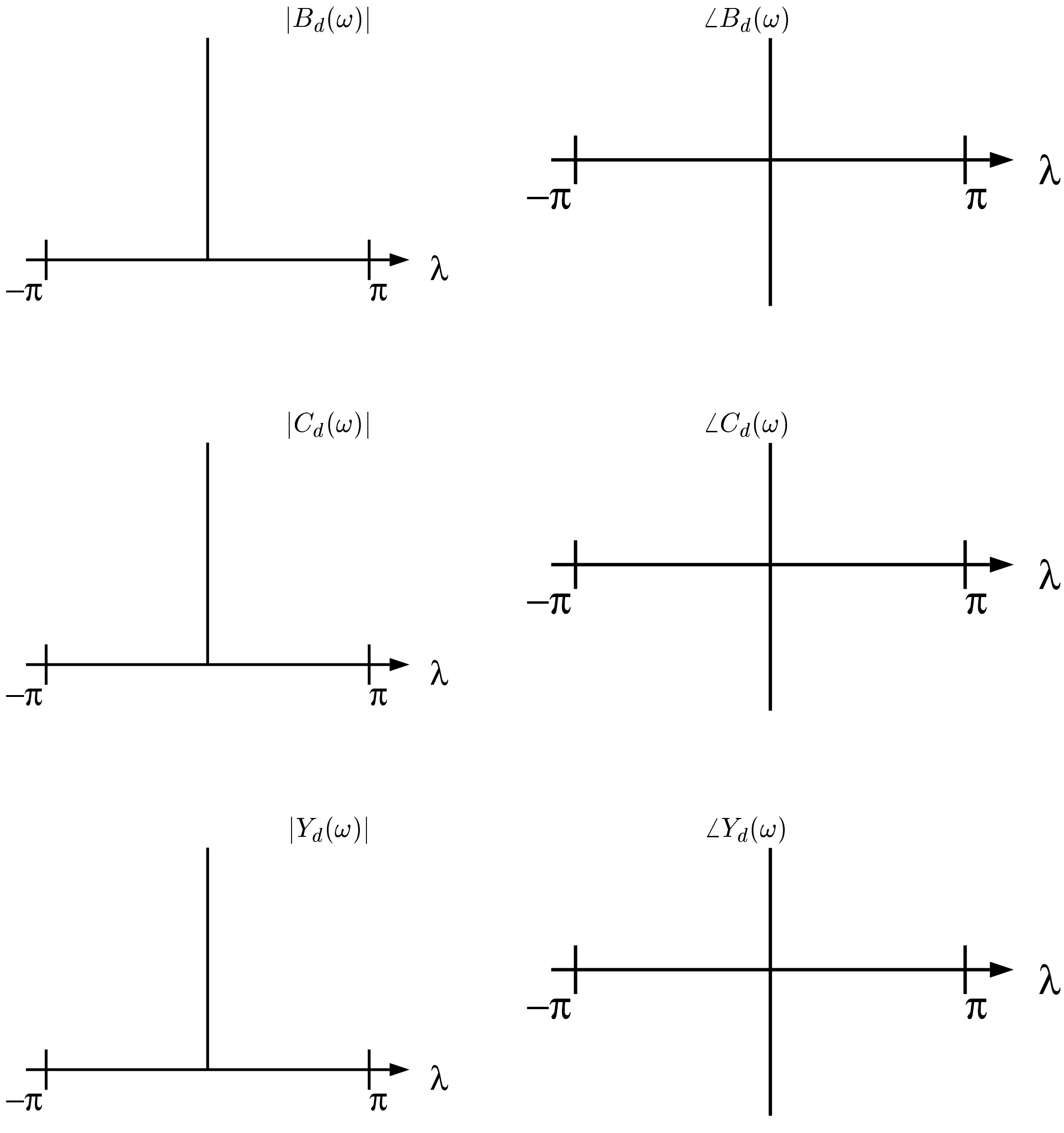
$$H_1(\omega) = \begin{cases} 1, & |\omega| \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < |\omega| < \pi \end{cases} \qquad H_2(\omega) = e^{-2j\omega}, \quad |\omega| < \pi$$



(a) For each of the signals, $a[n]$, $b[n]$, $c[n]$, and $y[n]$, plot the magnitude and phase of their DTFT below (1 point each).



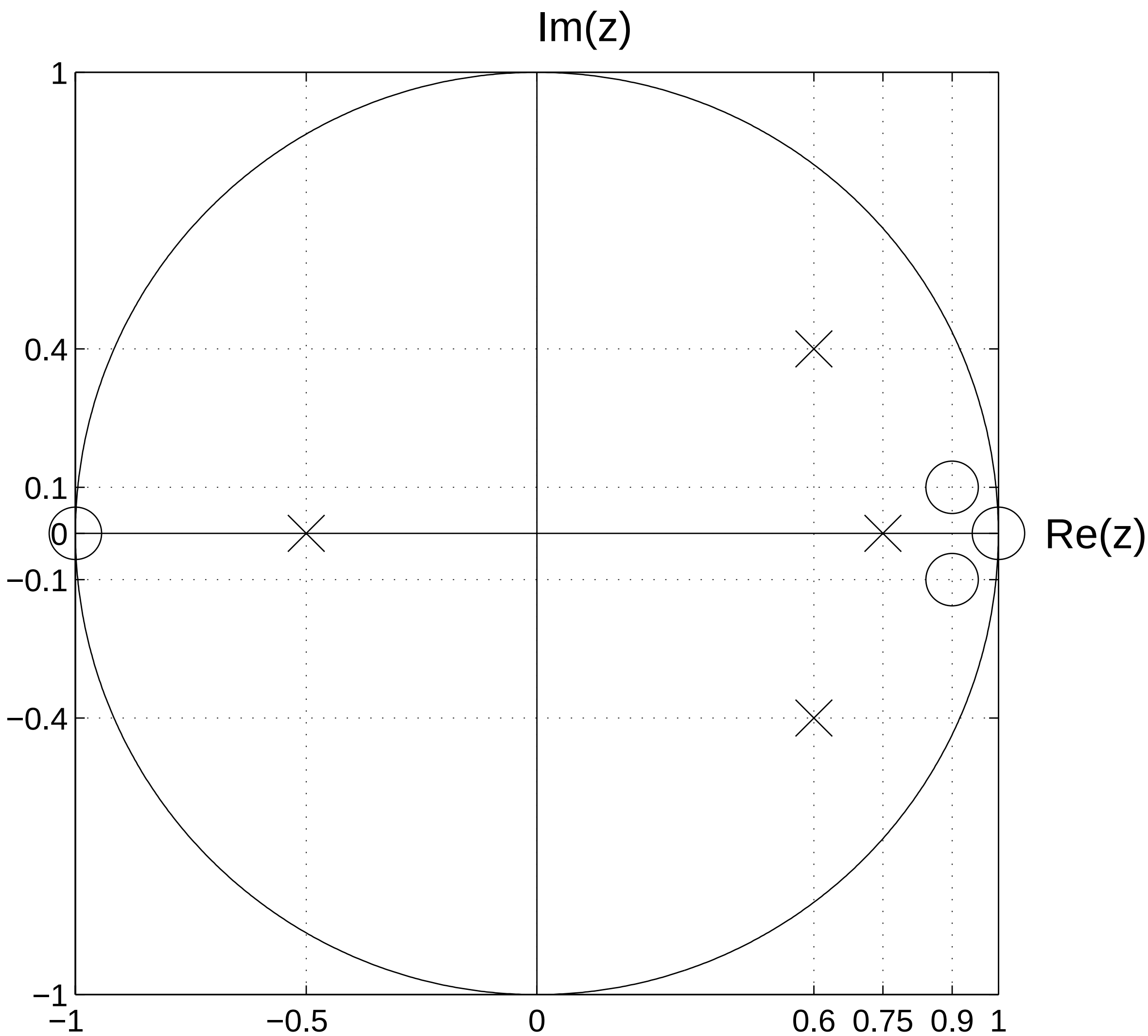




(b) Determine the sequence $y[n]$. (2 points) $y[n] =$ ______________________________

Problem 4 (10 points)

A causal, stable discrete-time filter with transfer function $H(z)$ has poles and zeros as shown in the following figure:



Pole-zero plot

and satisfies, $\lim_{|z|\to\infty} H(z) = 2$. On the following page, draw a block diagram for a possible implementation of $H(z)$ using a cascade of two second-order subsections, where each subsection is in direct form I and has real coefficients.

Draw your implementation of $H(z)$ here, and indicate your reasoning.